Foundations of Mathematics – blended learning design

Case study

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Introduction

This report is about the development of the case study in mathematics at Brno University of Technology (further referred to as "BUT"). We report on the background, team, context, design, and implementation of a blended version of the course Foundations of Mathematics with learning activities implemented in a learning management system (further referred to as "LMS").

Motivation and goals

The goal of the case study was to design educational activities that can be carried out under various circumstances. Main motivation for this goal came with the COVID-19 pandemics and related safety measures and restrictions (e.g. lockdowns, quarantines) that had significant impact on the classroom-based activities and students' learning in general.

The motivation for the redesign of the selected course was the growing need for supporting students in recalling high school mathematics and helping them understand the content of mathematics courses in the first year of their study. The main goal of the design was to motivate the students to practice mathematics regularly and work during the whole semester. Another goal was to give them opportunity to talk about mathematics and learn how to express their thoughts about mathematical topics and thus foster their understanding of mathematics.

Background of the case study

BUT is located in Brno, the capital city of the Southern Moravia region, and provides engineering education to students since 1899. Currently it has about 1170 academic employees who take care about more than 18 000 students distributed among 8 faculties and 2 research institutes.

The Central European Institute of Technology (CEITEC), where the BoostEdU project was managed, constitutes the key element of a world-class research infrastructure providing state-of-the-art equipment and ideal conditions for basic and applied research, especially in material science.

Developmental activities within the BoostEdU project took place at the Department of Mathematics in the Faculty of Electrical Engineering and Communication (DM FEEC). The

Department of Mathematics takes care of mathematics teaching for two faculties, the Faculty of Electrical Engineering and Communication and the Faculty of Information Technology. Mathematics courses are usually organised for cohorts with large numbers of students, ranging from 100 to 900 students with various backgrounds.

Course rationale

The course chosen for the digital redesign was the one with the internal code “BPC-MAS” and the title “Mathematical seminar”. An alternative translation into English might be “Foundations of Mathematics” as the purpose of the course is to help the students to recover and deepen their knowledge of high school mathematics topics. It is one of the three courses of the type “seminar” where no grades are awarded, it is just passed/did not pass. Each student must pass at least one course of the three courses offered every year. Contents of the course includes foundations of mathematics, equations, inequalities, vectors, elementary functions, limits, differentiation, complex numbers, sequences, infinite series.

Primary target group is the cohort of the first-year bachelor students of electrical engineering at the Faculty of Electrical Engineering and Communication. However, second-year or third-year students can eventually sign up too in case they need it to fulfil the condition of passing one of the three courses of the type “seminar”.

Teachers engaged in the course typically are internal doctoral students who must complete certain number of teaching hours during their study, and external teachers including PhD students at other faculties or universities.

Before the redesign, the course was organised as fully classroom-based, with mandatory participation of students and one or two written tests. The number of points from the two tests were summed up and students had to reach certain score to pass the course. Moreover, the topics covered in the course used to contain integration methods as well.

During the project implementation period, we had the following numbers of course participants. There were 411 students registered in the course in the beginning of the fall semester 2021. They were split into 6 groups of almost 70 students. In the beginning of the fall semester 2022, there were 453 students registered in the course. They were split into 6 groups of about 75 students.

Methodology

During the case study preparation and implementation, the following methodologies have been used: blended learning, design thinking, active learning, game-based learning, peer assessment/feedback, flexible learning.

The general methodology for the case study – blended learning (Wikipedia) – was chosen based on the project call "Digital Education Readiness" and the focus of the project intellectual output "Digital Education Methodology". For the course redesign and update, design thinking methodology has been used (Wikipedia). Learning activities have been developed based on active learning (Freeman 2014), game-based learning (Top Hat glossary) and peer assessment & feedback (Hattie 2007, Reinholz 2018). The course organisation was set up in line with principles of flexible learning (Shurville 2008, Top Hat glossary).

During the implementation and sustainability phase of the project, the following tools have been used or suggested for use: LMS Moodle, Workshop activity, Quiz activity, MS Teams, MS OneNote, MS Forms, MS Stream / YouTube, Wikipedia, mathematical and computational tools (Wolfram Alpha, PhotoMath, GeoGebra, Desmos, Symbolab, Matlab, Octave, MathsTools).

The course design has been implemented in the LMS "Moodle" which is the e-learning platform used by BUT. Learning activities were implemented through the Moodle built-in activities "Workshop" and "Quiz".

Moodle is a free and open-source learning management system written in PHP and distributed under the GNU General Public License. Moodle is used for blended learning, distance education, flipped classroom and other online learning projects in schools, universities, workplaces and other sectors (Wikipedia).

The Quiz is a very powerful activity that can meet many teaching needs, from simple, multiple-choice knowledge tests to complex, self-assessment tasks with detailed feedback. Questions are created and stored separately in a Question bank and can be reused in different quizzes. When creating a Quiz you can either make the questions first and add them to the Quiz, or add a Quiz activity and create the questions as you go along (Moodle).

Workshop is a powerful peer assessment activity. Students add submissions which are then distributed amongst their peers for assessment based on a grading scale specified by the teacher (Moodle).

More details about the LMS can be found on the Moodle website: https://moodle.org

MS Teams and MS OneNote were used as collaborative platforms for the course design, learning activities content design and feedback questionnaires design.

MS Forms was used to develop questionnaires for collecting student feedback to the learning activities and the entire course.

Students were suggested to use video hosting platforms like MS Stream or YouTube to upload their own works. They also were encouraged to use a variety of free mathematical and computational tools and apps that can be used both in computers and mobile phones to help them in solving mathematical tasks. Wikipedia and streaming platforms were suggested as complementary study resources.

Design (2021)

The first step in the preparatory phase was building a team. The idea was to bring in people interested in helping the students to recover or complement their knowledge of high school mathematics. The development team consisted of five people and included one internal and two external teachers, a former PhD student and a colleague from outside the academia. All team members have met before in the international educational project "Partnership for Learning and Teaching in University Mathematics" with acronym PLATINUM co-funded by the EU, Grant Agreement Number 2018-1-NO01-KA203-038887 (PLATINUM book, 2021).

Considering that the main goal of the design was to motivate the students to practice mathematics regularly and work during the whole semester, the team members discussed how such a blended learning design could look like. Inspiration for one part of the design was taken from the UK-based platform "Teaching and Learning Mathematics Online" (TALMO). In the presentation by Mark MacDonald, a significant increase in student activity and performance was reported when they were given a sequence of quizzes consisting of several multiple-choice questions with unlimited number of attempts. Inspiration for the other important part of the design – recording and evaluating videos – stemmed from own experience of the team members as internet users. First, videos might be a useful source of information and knowledge, and certain level of understanding is required for recording a useful video. Second, ratings and reviews of products and services help people decide what they will buy/use/watch.

Based on the discussions, decision was made that the course design should have two components: classroom activity with scheduled teaching, and e-learning part with activities scheduled in LMS Moodle. To pass the course, students must complete the e-learning component while the classroom component should be non-graded. A dedicated team was created in MS Teams, and that team's OneNote notebook was used to outline the course agenda and develop contents of each scheduled LMS activity. As none of the team members had a detailed prior knowledge of LMS Moodle, available activities were investigated, and convenient options were chosen. The activity "Test" which is the Czech form of the Quiz activity was chosen for implementation of quizzes, while for collecting and evaluating video recordings the activity "Workshop" was found optimal.

Two sub-teams were set up, one responsible for developing the database of tasks and setting up quizzes, the other responsible for preparing a list of topics for video recording and

setting up workshops. The process was incremental and iterative. The tasks were prepared, and quizzes set up sequentially during the autumn 2021. Similarly, the lists of topics for video recording were developed and workshops set up one after another during the semester.

A workshop design consisted of:

- a list of topics with detailed description of what should be covered in each topic;
- lists of students and the topics assigned to them;
- arrangement and management of the "workshop" activity.

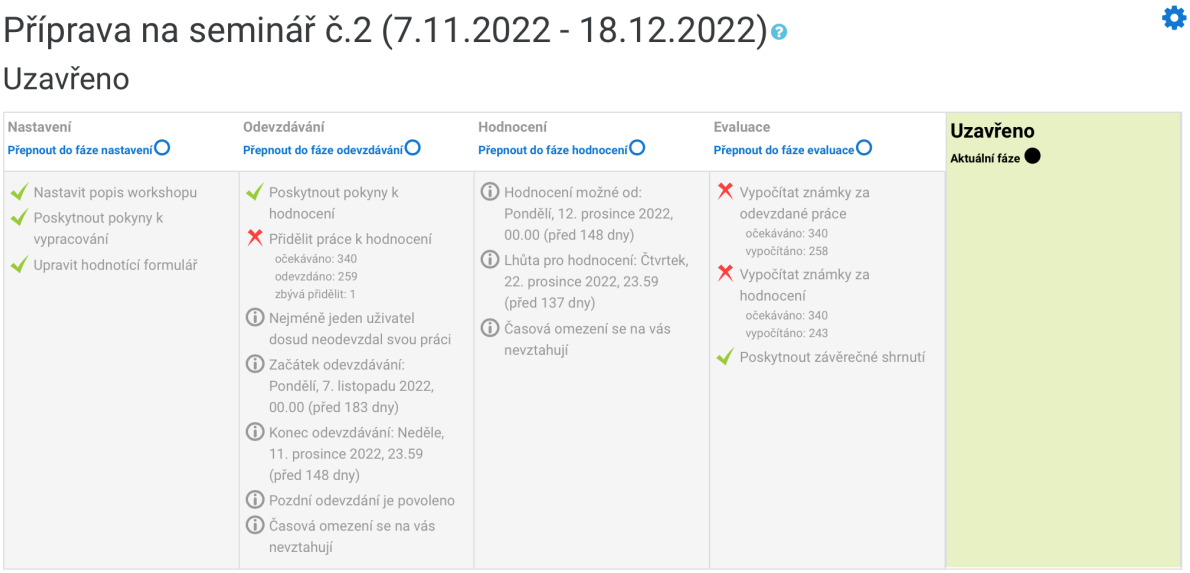

Figure 1 Workshop "dashboard" from the teacher's point of view.

A quiz consisted of:

- a folder of questions in the Question Bank related to the topic of the quiz;
- setting up the quiz and the period when the quiz should be available.

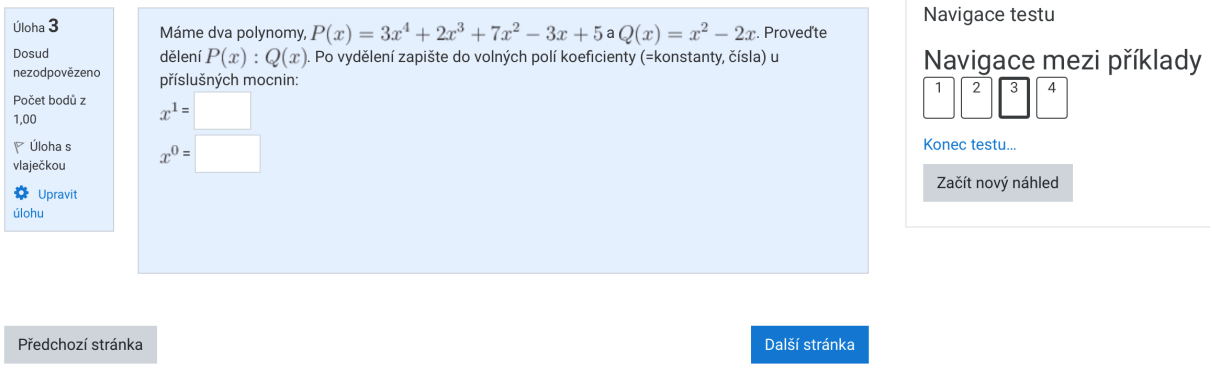

Figure 2 Sample of a question with navigation buttons and menu.

For the classroom activity, the teachers had freedom to choose the way how they wanted to teach a particular topic, in the sense that there were no specific lists of theorems or procedures that must be taught. They were suggested to give a brief overview of the topic, recall important formulas, and demonstrate the use of them on a couple of examples. After such a teaching activity, a list of tasks should be presented to students, and they should work on the tasks with the support of the teacher.

In the autumn 2021, the course was implemented with both components, classroom activities and the e-learning part realized in LMS Moodle. The LMS part was compulsory in the sense that it was necessary to finish all given tasks to complete the course. Participation in the classroom exercises was optional. The e-learning part consisted of 4 workshops and 13 quizzes. In each of the workshops, a student had to record and made available a short video (up to 5 min) on the assigned topic, and to evaluate three videos prepared by other students. The schedule for each workshop was three weeks for recording and one week for evaluation, where the evaluation period was overlapping with the first week of the recording period of the following workshop. Each quiz contained between 5 and 12 questions (tasks) chosen randomly from specified folders in the Question Bank. One new quiz was introduced every week of the semester. The deadline was set common to all quizzes: the end of the exam period that followed the semester. To complete a quiz, 100% of correct answers was required, therefore the number of attempts to complete the quiz was not limited. To prevent students from guessing answers by running one attempt right after another, after two non-successful attempts there was a forced 90min break before any other attempt.

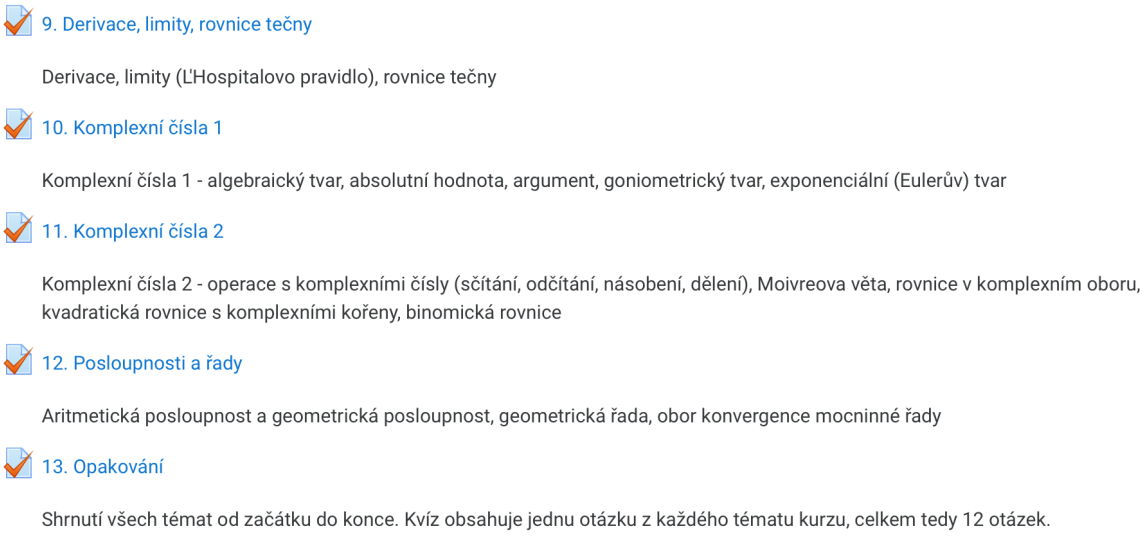

Figure 3 Sample of the quizzes arrangement in 2021.

The role of a teacher in this arrangement was two-fold. First, they should provide students with information about the topic and give space to ask questions during the classroom activity. However, the former classroom testing was removed and replaced by online activities. The second role of the teacher was thus to follow students' progress through quizzes and encourage them to work regularly. The teacher should also check whether the

students uploaded their videos to the workshops and evaluated the videos of other students afterwards.

During the semester, the quizzes ran smoothly except that a high number of typos, mistakes and errors was reported. The tasks were updated whenever such a situation happened. We anticipated this in the first round of implementation as there were more than 1000 questions developed by hands of human beings. Things did not run so smoothly with the workshops, deadlines for both video submission and evaluation had been extended several times upon students' requests. Some of the students decided to complete the course even after they were done with exams. In the end, approximately 56% of all students completed the course successfully.

Feedback (2021)

At the end of the autumn semester 2021, the students were encouraged to fill in a feedback questionnaire where they were asked to answer questions and give comments about the following topics:

- General information – previous education, time spent weekly on mathematics during the semester.
- Difficulty, positives/negatives of the quizzes, videos and classroom activity.
- Optional comments to quizzes, videos and classroom activity.
- Overall evaluation of the course.

In total, 108 students responded to the questionnaire. Such a high number was achieved by offering students bonus points that could be used to finish incomplete quizzes.

Students found positive about the quizzes repetition and practice, varied and inventive tasks, entertaining form of learning, unlimited number of attempts and time, instant feedback (displaying correct answer and links to learning resources), verification of knowledge, possibility to work when one wants to work, made them spend more time on mathematics, visible self-improvement, need for complex thinking.

Some students reported that the quizzes were time consuming and difficult to complete without mistakes.

Tips from students for improving the quizzes:

- Decrease the required 100% proportion of correct answers.
- Make shorter quizzes with less questions.
- Remove the 90min break before the next attempt.

- Repeating only the questions with wrong answers.
- Remove errors and mistakes.
- Formulate tasks more clearly.
- Provide sample solutions.

Regarding workshops, students often emphasized that it was necessary to study the topic more deeply and understand it well, in order to explain it to others. They also appreciated the opportunity to listen to explanations of other classmates with different perspectives/points of view.

Tips from students for improving the workshops:

- Unify recording medium.
- Improve access to videos of others.
- Less workshops/recordings per semester.
- Required length of a video could be shorter.
- All topics could be given in the beginning.
- Evaluation period could be longer.

Students liked about the classroom activity division to two parts (explanation and practice), comprehensible and well-structured explanation, slower pace, individual approach, relaxed environment, voluntary participation, forthcoming teacher.

Three questions concerned the benefit of particular parts of the design (quizzes, video recording, video evaluation):

- 4. Did solving quizzes help me to understand mathematics better? (Q – quizzes)
- 8. Did the workshops (recording and evaluating videos) help me to understand mathematics better? (W – workshops)
- 12. Was it beneficial for me to watch and evaluate videos recorded by peers? (E – evaluation)

The following table summarises students' answers (%):

	Yes	No	Other
Q	52,78	32,41	14,81
W	50,00	41,67	8,33
E	49,07	43,52	7,41

Intersections and unions of the groups of students that answered “Yes” to the questions (%) are reported in the following table:

	Intersection	Union
Q W E	25,00	76,85
Q W	29,63	62,04
Q E	37,04	60,19
W E	38,89	58,33

We can observe that 25% considered all three activities (quizzes, making videos, video evaluation) beneficial for their learning. On the other hand, more than 75% considered at least one activity as beneficial.

Summary and conclusions (2021)

- Majority of the students (75%) mentioned that the digital part of the course helped them to understand mathematics better.
- 56% students finished the course successfully.
- Main obstacle: difficulty level of the quizzes.
- Students had the whole semester for each quiz to complete.
- Workshops were time constrained.
- Form and content of the classroom activity was teacher dependent.
- Significant part of the students (about 25%) felt that more time should have been given to topics in the parallel course BPC-MA1 Mathematics 1 (Calculus 1).

Hypotheses and open questions (2021)

- How to reach out to the remaining 25% of students?
- Next time we include a question “Did the classroom activity help you ...”.
- Quizzes difficulty: would it help to split the quizzes into smaller ones?
- Quizzes should be time constrained.
- Reduce the number of workshops from 4 to 2 and make the students go deeper.

Unexpectedly, the course implementation in the autumn 2021 had impact on the departmental scale. The colleague responsible for the preparatory course in mathematics targeting high school students decided to implement the course online as a combination of recorded videos and quizzes on selected topics from our course Foundations of Mathematics. The decision was made to split the quizzes into shorter ones. After the successful conclusion of the preparatory course, the same decision was made for Foundations of Mathematics.

(Feedback-informed) redesign (2022)

In the autumn 2022, we found out that we did not remember how to do certain procedures, for example import groups and show them in the Grades dashboard. We had to rediscover those processes and this time we recorded it by making notes in the team OneNote notebook.

The course was also implemented with both components, classroom activities and the e-learning part realized in LMS Moodle. The LMS part was compulsory in the sense that it was necessary to finish all given tasks to complete the course. Participation in the classroom exercises was mandatory but non-checked. The quizzes and topics for the video recording were arranged differently than in the autumn 2021. We split the quizzes into smaller ones and reduced the number of workshops. We did not have time to modify or exclude anything from the workshop topics. The e-learning part then consisted of 2 workshops and 40 mandatory + 2 voluntary quizzes. In each of the workshops, a student had to prepare material on the assigned topic, and to evaluate three materials prepared by other students. Originally, we wanted students to record videos as well as in the autumn 2021, but due to an unexpected pressure from some department members we were forced to continue with a milder request of a “material” instead of a video. The schedule for each workshop was six weeks for the material preparation and one week for evaluation, where the evaluation period was overlapping with the first week of the preparation period of the following workshop. Each quiz contained between 2 and 5 questions (tasks) chosen randomly from specified folders in the Question Bank. Between 2 and 4 quizzes on a new topic were introduced every week of the semester. Quizzes in the same topic had the same deadline, which was 2 weeks after the quiz was introduced. Again, to complete a quiz, 100% of correct answers was required, therefore the number of attempts to complete the quiz was not limited. To prevent students from guessing answers by running one attempt right after another, after two non-successful attempts there was a forced 15min break before any other attempt. In principle, this modified design meant that all activities should had been finished before the exam period started.

Figure 4 Sample of the quizzes arrangement in 2022.

During the semester, the quizzes ran smoothly except that a few typos, mistakes, and errors were reported (less than 10). The biggest mistake, which unfortunately was not reported, happened close to the end of the semester. One of the topics that should contain three different quizzes with 2 (demanding) questions by mistake contained three identical quizzes with 6 (demanding) questions. This only turned out in the feedback questionnaire, and it was too late to do anything about it. Again, we had difficulties with workshops, the deadlines for both video submission and evaluation had been extended several times upon students' requests. Moreover, because of the unclear instructions about the form of the material, some students submitted just photographs of some calculations in bad quality which was not appreciated by their peers. In the end, again approximately 56% of all students completed the course successfully.

Feedback (2022) and comparison to 2021

At the end of the autumn semester 2022, the students were encouraged to fill in a feedback questionnaire with the following content:

- General information – previous education, time spent weekly on mathematics during the semester.

- Difficulty, positives/negatives and helpfulness of the quizzes, videos AND classroom activity.
- Optional comments to quizzes, videos and classroom activity.
- Overall evaluation of the course.

In total, 30 students responded to the questionnaire. This time we did not offer students bonus points that could be used to finish incomplete quizzes.

In the following figure, we can see comparison of helpfulness of quizzes, workshops and evaluation of videos/materials between 2021 and 2022.

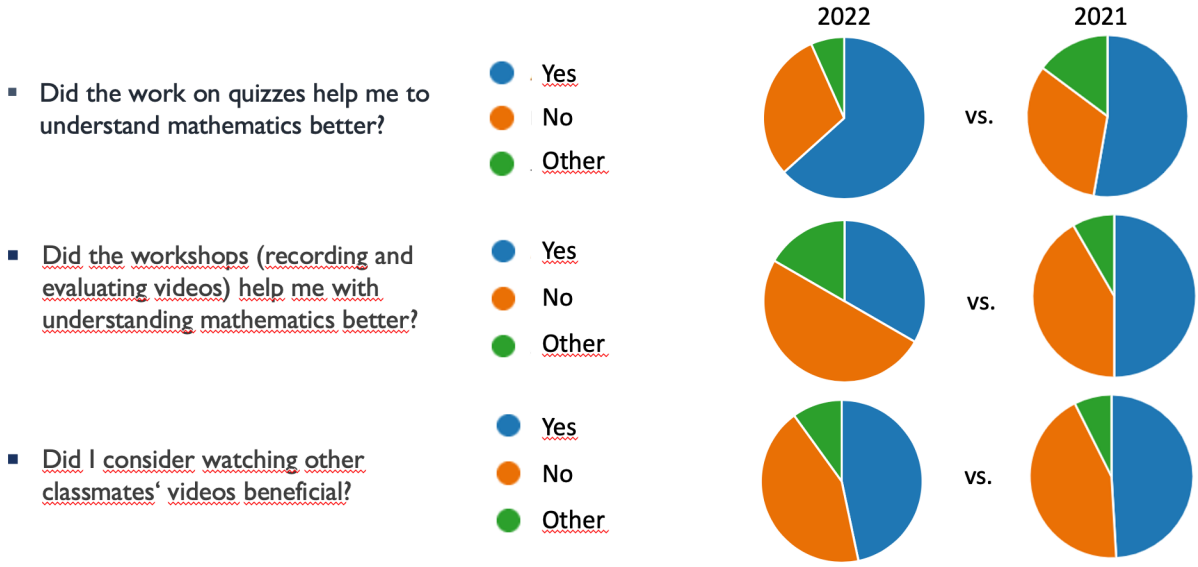

Figure 5 Comparison of helpfulness of LMS learning activities between 2021 and 2022.

We can observe two phenomena. First, the improved quiz design (splitting into smaller pieces) had positive effect and larger proportion of students in 2022 considered the work on quizzes as helpful for understanding mathematics better. Second, the ambiguity of instructions about the form in which the workshop topics should be submitted had negative effect and larger proportion of students in 2022 considered the workshops less helpful, even though they had to submit only 2 works which was much less compared to 4 videos that students in 2021 had to record.

This time, four questions concerned the benefit of particular parts of the design (quizzes, video recording, video evaluation, classroom activity)

- 4. Did solving quizzes help me to understand mathematics better? (Q – quizzes)
- 8. Did the workshops (recording and evaluating videos) help me to understand mathematics better? (W – workshops)

- 12. Was it beneficial for me to watch and evaluate videos recorded by peers? (E – evaluation)
- 17. Did the classroom activity help me to understand mathematics better? (C – classroom)

Intersections and unions of the groups of students that answered “Yes” to the questions (%) are reported in the following table:

	Intersection 2021	Intersection 2022	Union 2021	Union 2022
Q W E C		23,33		90
Q W E	25,00	26,66	76,85	83,33
Q W	29,63	30	62,04	66,66
Q E	37,04	30	60,19	80
W E	38,89	26,66	58,33	53,33

We can observe that 23,3% considered all activities (quizzes, making videos, video evaluation, classroom activity) beneficial for their learning in 2022. On the other hand, 90% considered at least one activity beneficial.

Summary and conclusions (2022)

- Majority of the students (83,33%) mentioned that the digital part of the course helped them to understand mathematics better, which means improvement compared to 75% in 2021.
- Again, 56% students finished the course successfully.
- Main obstacle: time required to complete the quizzes.
- Students had limited time for each quiz to complete (2 weeks).
- Workshops were time constrained as well, but with significantly more time for preparation of the material (6 weeks compared to 3 weeks in 2021).
- Form and content of the classroom activity was again teacher dependent.

Experience & lessons learned

We learned a lot of experience during the two years of work on the course Foundations of Mathematics.

First, regarding the classroom activity, we saw that there is no big difference between non-mandatory and mandatory-but-not-checked. On the other hand, the blended learning design was convenient for various circumstances and gave students more freedom and flexibility, for example in case of illness or quarantine.

In both years, majority of the students mentioned that the digital part of the course helped them to understand mathematics better (77% resp. 83%). After the improvements implemented in the second year, 90% of the students mentioned that the course helped them to understand mathematics better. This suggests that shorter, time-constrained quizzes support students' learning better. It was also interesting to see that about 30% of active students tried the first voluntary quiz concerning limit of a function in 2022, suggesting that the students were eager to learn, or at least curious enough to try.

One thing that was clearly visible from the data from the collected feedback was that the medium to deliver the workshops' outputs should be uniform. There was less complaints and more positive evaluation regarding the workshops in the first year when the requested medium was uniform, even though the workload was much higher than in the second year.

In both years, 56% of enrolled students completed the course successfully. However, about 86% of students who completed the course in 2021 continued to study one year after completing the course, which is significantly higher proportion than for students who did not complete the course (51%) or who registered for the course but did not even try (19%). It suggests that there might be some correlation between completing the Foundations of Mathematics course and students' resilience.

Similar phenomenon was observed in the second year (2022) in terms of quality of "materials" submitted in the workshops. One month after the end of the course, there was about 56% of submitted works in the form of a video (29% of which were evaluated as good), 28% in the form of a photographed handwritten text, PDF of slides, and 16% were not accessible. Five months after the end of the course, there was about 88% of submitted works in the form of a video (72% of which were evaluated as good), 8% in the form of a photographed handwritten text, PDF of slides, and 4% were not accessible. The proportions were changing as students were dropping out, which again suggests that there might be some correlation between submitting a good quality material and students' resilience.

Challenges

One important challenge we see in coming to agreement with teachers and explain the philosophy to them repeatedly. The reason is that teaching duties are distributed by the Head of Department and consequently teachers in the course may differ year from year. One improvement that we plan to do is to prepare a feedback questionnaire or structured interview for teachers and ask them what would help them in this respect. Another activity that might help might be recording short videos with information for students about the course and tutorials about the quizzes and the workshops.

Another challenge that we are aware of is the students' (lack of) motivation to regular work. Electrical engineering students are not very fond of learning mathematics, and we need to explore why it is so through questionnaires and interviews.

A big challenge is the different level of maturity of the first-year students. However, thank to the colleagues from BoostEdU project we got an idea that might help us to address this challenge: let them work in pairs, or small groups.

Conclusions

First, we can conclude that the concept of the course Foundations of Mathematics redesign and implementation were successful in the mail goal: to support first-year students in helping them understand mathematics better. During the two cycles of design thinking procedure, we saw an improvement in students' evaluation of the course thank to the adjustments in the course design. Higher proportion of students mentioned that the course helped them in understanding mathematics in the second run.

Further, we can say that also the goal of the case study – to design educational activities that can be carried out under various circumstances – was achieved. The implemented design provided students more flexibility for their learning, especially in cases when they got sick or had to stay in quarantine.

Finally, we saw that the ideas we used in the design had impact on teaching and learning activities in other courses at BUT and beyond. The quiz design has been optimised and implemented in the preparatory course in mathematics at BUT. The idea of peer evaluation/feedback was adapted for use in several courses at Masaryk University in Brno.

We appreciate the opportunity to work on the digital redesign of the course and to collect valuable experience. We plan to use this experience for further optimisation of the course design, for example for extending the Question Bank, but also for digitalisation of other courses.

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